Optimized output-based input shaping for control of single-link flexible manipulator using linear matrix inequality

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ABSTRACT

Precise hub angle positioning due to tip deflections, flexible motions and under various payloads is enormous tasks in the control of single-link flexible manipulators. In this paper, output-based command shaping (OBCS) was designed using the system output for tip deflections and residuals vibrations suppression, and this was incorporated with a linear matrix inequality (LMI) closed-loop control scheme for precise hub angle positioning. The robustness of the hybrid control scheme was tested by changing the payloads from 0g to 30g, and 50g. Simulation results showed that endpoint residuals vibrations and tip deflections due to flexible motions were suppressed and hence precise hub angle positioning under various payloads was achieved. Integral absolute error (IAE), Integral square error (ISE) and Time response analysis (TRA) were used as the performance indexes. Hence, the hybrid control scheme is simple and robust.

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1. INTRODUCTION

The flexible manipulator is an equipment used in transporting objects from one position to another, which find applications in automotive industries, space exploration, nuclear plants, and aerospace. The flexible arm comprises of multiple parts that are controlled electronically in performing various industrial tasks such as welding and assembling of parts [1–4]. Flexible manipulators are categorized into flexible and rigid body manipulators. Compare to rigid body manipulators, flexible manipulators are safer and more economical, and they have high speed, less mass, and less energy consumption [5]. Recently, the control of flexible manipulators has attracted the attention of many researchers. Several control algorithms such as feedforward and feedback control methods were proposed and employed in the control of a single-link flexible manipulator. The feedforward control method is applied for residual vibration control while the manipulator coordinate is mainly controlled using the feedback control approach. Modern control strategies such as robust and adaptive control methods were employed to control the single-link flexible manipulators [1, 4, 6]. Furthermore, studies [6-9] suggested zero vibration (ZV), zero vibration derivatives (ZVD), and zero vibration derivative derivatives (ZVDD) to suppress residual vibrations and oscillations. In [10], a hybrid control schemes for tip deflection suppression and hub angle tracking was proposed. Where the
input command filter was designed to suppress tip deflection and was incorporated with LQR for tracking control. Researchers [11] proposed proportional integral derivative (PID) as feedback control for rigid body motion control of single-link flexible manipulator. A hybrid output based input shaping control scheme for vibration and position control of a single-link flexible manipulator was proposed in [12], an output-based filter was designed to reduce tip deflection while the linear quadric regulator (LQR) controller and PID controller were incorporated for precise hub angle positioning of the single-link flexible manipulator.

Besides, other researchers’ have proposed various hybrid control schemes for flexible manipulators. The adaptive interactive proportional-derivative (PD) control approach for active vibration and position control of the pneumatic drive was investigated in [13]. The system’s rigid body motion was controlled using the pneumatic drive system. In [14], input tracking of flexible manipulators using a PD, PID, ZVDD and a hybrid fuzzy control approach was investigated. LQR, LQ-PID, and hybrid LQR-Input shaping methods were proposed in [15] for hub angle control and residual vibration suppression of the flexible manipulator systems. A non-collocated PID and LQR controllers for vibration and input tracking control was implemented in [16]. S. Dei et al [17] presented a tuned PD controller optimized using a modified genetic algorithm (GA) for vibration and input tracking control of flexible manipulator. Simulation studies in [18], demonstrated the capability of feedback linearization and input shaping control approach in the elimination of residual vibrations. The robust control technique was presented [19] and Research in [20] suggested a nonlinear adaptive predictive model controller for vibrations and position control of single-link flexible manipulator. Shape optimization of the revolute joint of a single-link flexible manipulator for suppression of vibrations was presented in [21]. Adaptive control of a single-link flexible manipulator using the sliding mode technique was presented [22], and Active control of a single-link flexible manipulator using nonlinear conventional motor was proposed in [23], and the two nonlinear controllers were presented for tip deflection suppression and hub angle tracking control of the system.

Owing to the presented literature there is still the need to increase the precision in hub angle positioning via a simple and cost-effective control scheme to achieve accurately and effectiveness of pick and place of an object in the industries. This paper proposed a novel hybrid control scheme, where output-based command shaping filter was incorporated with LMI control techniques for tip deflections suppression and precise hub angle control of single-link flexible manipulator. An output-based command filter was designed using the system output to suppress tip deflection while the LMI closed-loop control schemes were employed for hub angle tracking control and optimization. The system dynamics, controller design, and results and discussion are presented in Section 2, 3 and 4 respectively while Section 5 presented conclusions and future recommendation.

2. SYSTEM DYNAMICS

The single-link flexible manipulator system is as shown in Figure 1, whereas Figure 2 shows the schematic diagram of the system. The mathematical model representation of the system is obtained using the finite element method. The system comprises of a non-movable coordinate X0Y and a movable coordinate POQ. And the applied torque at the hub is represented by τ (t). The parameters Mf, V, E, I, P and Ih are manipulator payload, area moment of inertia, young modulus, mass density per unit volume, cross-sectional area and hub inertia respectively [24]. Table 1 shows the system parameters. The effects of rotary inertia and transverse shear are neglected as the system is long and slender and only moves in the XOY plane. Based on the aforementioned assumption, the Bernoulli-Euler beam theory is used in developing the model for the system’s elastic behaviour. Moreover, the system is assumed to have a constant cross-sectional area, uniform material property, and continuous vibrations in the horizontal direction with negligible gravitational effect and experience torsion and is stiff in vertical bending. The single link manipulator system is made of aluminium material with dimension 900 mm x 19.008 mm x 3.2004 mm [25].

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbols</th>
<th>Values</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young modulus</td>
<td>E</td>
<td>71x10^3</td>
<td>N/m^2</td>
</tr>
<tr>
<td>Density per unit volume</td>
<td>ρ</td>
<td>2710</td>
<td>Kg/m^3</td>
</tr>
<tr>
<td>The second moment of inertia</td>
<td>I</td>
<td>5.1924</td>
<td>m^4</td>
</tr>
<tr>
<td>Flexible link length</td>
<td>L</td>
<td>0.96</td>
<td>m</td>
</tr>
<tr>
<td>Flexible link width</td>
<td>W</td>
<td>0.019</td>
<td>m</td>
</tr>
<tr>
<td>Flexible link thickness</td>
<td>B</td>
<td>0.003</td>
<td>m</td>
</tr>
<tr>
<td>Hub inertia</td>
<td>Ih</td>
<td>5.86 x 10^-4</td>
<td>Kgm^2</td>
</tr>
<tr>
<td>Moment of inertia</td>
<td>Ih</td>
<td>5.1924</td>
<td>Kgm^2</td>
</tr>
</tbody>
</table>

Table 1. System parameters [25]
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The single-link flexible manipulator model used in this work is as obtained in [8, 24], the mathematical model representation of the system is obtained using the finite element method. Hence the state-space model was obtained as:

\[ \dot{\theta} = A \theta + B u \]  
\[ y = C \theta + D u \]

Thus, the A, B, C, and D matrixes are obtained as:

\[
A = \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 58209 & 0 & -33 \\
0 & -38548 & 0 & -27
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
0 \\
0 \\
1013.6 \\
-821.0
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
1 & 0 & 0 & 0
\end{bmatrix}
\]

\[
D = \begin{bmatrix}
0
\end{bmatrix}
\]

\[
\theta = \begin{bmatrix}
\theta \\
\alpha \\
\dot{\theta}
\end{bmatrix}
\]

The matrix \( \theta \) represented the state variables, which are hub angle and tip deflection respectively and their derivatives.

3. CONTROLLER DESIGN

3.1. Output-based command filter

In this section, the output-based command filter was designed using the output of the single-link flexible manipulator. First, the reference system was designed based on the system dynamic response, and then system output was decomposed and measured using simulation. MATLAB codes formulated based on the mathematical model of the filter were used to obtain the filter gains. The basic principle of the output-based command filter strategy is presented by using the second-order system as expressed by (3) [26, 27].

\[ G(s) = \frac{k m w_m^2}{s^2 + 2 \zeta w_m s + w_m^2} \]  

(3)

The reference system was set to be designed as:

\[ G_r(s) = \frac{k_m w_m^2}{s^2 + 2 \zeta w_m s + w_m^2} \]  

(4)

Also, the filter can be designed as:

\[ F(s) = \frac{k_m w_m^2 s^2 + 2 \xi w_m s + w_m^2}{k_m w_m^2 s^2 + 2 \xi w_m s + w_m^2} \]  

(5)

Thus, Product of G(s) and F(s) will yield G_r(s) based on the principle of pole-zero cancellation, and hence, by selecting \( k_m, \xi, w_m \), adequate static gain, damping ratio, and bandwidth can be realized respectively [28].
Hence;

\[ F(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + 2 \xi \omega_n + \omega_n^2} \]  

(6)

The purpose is to get the filter gains \((a_0, a_1, a_2)\) so that zeros \(F(s)\) will cancel the identical poles of \(G(s)\) [28]. The critically damped system is considered in designing a reference system which can be realized as;

\[ G_r(s) = \frac{w_c^2}{(s + \omega_c)^2} \]  

(7)

The parameter \(\omega_c\), which is the system bandwidth, is carefully chosen based on the system time response. The goal of the design is to obtain accurate filter coefficients so that the system has zero or little vibration. The single-link flexible manipulator considered in this work is a type-\(I\) system, and therefore, \(a_0\) is zero and the system is reduced to the fourth-order. Hence, the reference system was obtained as in (8) by selecting \(\omega_c = 10\) based on (7).

\[ G_r(s) = \frac{10^6}{s^4 + 40s^3 + 600s^2 + 4000s + 10^6} \]  

(8)

Thus, the filter gains were obtained by running the MATLAB code formulated based on the form;

\[
\begin{bmatrix}
 a_1 \\
 a_2 \\
 a_3 \\
 a_4
\end{bmatrix} =
\begin{bmatrix}
 S_{11} & S_{12} & S_{13} & S_{14} \\
 S_{21} & S_{22} & S_{23} & S_{24} \\
 S_{31} & S_{32} & S_{33} & S_{34} \\
 S_{41} & S_{42} & S_{43} & S_{44}
\end{bmatrix}
\begin{bmatrix}
 S_{3} I_r \\
 S_{2} r \\
 S_{3} r \\
 S_{4} r
\end{bmatrix}
\]

Substituting the gains, the filter was obtained as;

\[ F(s) = \frac{0.7952s^4 + 14.457s^3 + 484.729s^2 + 5680s + 10^6}{s^4 + 40s^3 + 600s^2 + 4000s + 10^6} \]  

(9)

### 3.2. LMI controller design

Stability and transient response of linear systems depend on the locations of the poles in the complex plane, the left half-plane as in Figure 3 is the stability region [29]. Consider a linear dynamic system;

\[ \dot{X} = A_c X \]  

(10)

The Lyapunov theorem states that the system in (10) is said to be asymptotically stable once a real symmetric matrix \(P\) satisfies the following LMIs [29];

\[ A_c P + A_c A_c^T < 0, \quad P = P^T > 0 \]  

(11)

The LMIs in (11) provides the conditions for the stability of the system in (10). The LMI region is as presented in Figure 4.

All the poles of the system in (10) will be lying in the LMI region of Figure 4 if and only if there exists asymmetrical positive definite matrix \(P\) such that;

\[ A_c P + P A_c^T + 2a P < 0, \]  

(12)

\[ A_c P + P A_c^T + 2b P > 0, \]  

(13)

\[
\begin{bmatrix}
 -rP & cP + P A_c^T \\
 cP + A_c P & -rP
\end{bmatrix} < 0, \quad P = P^T > 0
\]  

(14)
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The LMIs in (12) and (13) represent the vertical strip, while the LMI in (14) represents the circle centred at \((c, 0)\) with radius \(r > 0\). The controller gains \(K\) and the reference input scaling factor \(N\) were found as \([0.873, 63.517, 0.2389, 0.1649]\) and \([0.873, 63.517, 0.2389, 0.1649]\) respectively by using MATLAB codes. The complete closed-loop control scheme block diagram is as shown in Figure 5.

![Figure 3. Left half-plane](image1)

![Figure 4. LMI region](image2)

The complete closed-loop control scheme block diagram is as shown in Figure 5.

4. RESULTS AND ANALYSIS

To precisely control the single-link flexible manipulator for set-point tracking, a bang-bang input torque was used as the reference angle. This evaluates the performance of the LMI-OBCS under various payloads for tip deflection suppression and tracking control. The simulation results of the proposed control schemes are presented in this section. The payload mass was changed from 0g to 30g, and 50g to test the robustness of the proposed novel hybrid control scheme to parameter variation. The optimized controller gains were obtained as; \(K = [0.873, 63.517, 0.2389, 0.1649]\). Figure 6 and Figure 7 show the tip deflection and hub angle under 0g payload. Thus, it can be observed that tip deflections were suppressed and precise hub angle tracking was achieved. The maximum tip deflections of 2.20 \(x10^{-3}\)m and settling time and rise time of 1.3 sec and 0.95 sec respectively with zero steady-state error and zero overshoot were observed.

![Figure 5. Block diagram of the closed-loop control scheme](image3)

![Figure 6. Tip deflection response to LMI-OBCS controller](image4)

![Figure 7. Hub angle response to LMI-OBCS](image5)
This shows that the objectives of the control schemes were achieved. Also, the proposed control schemes were tested under various payloads mass as shown in Figure 8 and Figure 9. From these figures, it is clear that no significant changes were observed under various payloads. Thus, the tip deflections changes under 30g from $2.20 \times 10^{-3}m$ to $3.3x10^{-3}m$ and settling time and rise time from 1.3 sec and 0.95 sec to 1.25sec and 0.85 sec respectively with no changes in steady-state error and overshoot.

![Figure 8. Tip deflection response to LMI-OBCS controller with payloads variation](image1)

![Figure 9. Hub angle response to LMI-OBCS controller with payloads variation](image2)

While under 50 g it changes from $2.20 \times 10^{-3}m$ to $3.9x10^{-3}m$ and settling time and rise time from 1.3 sec and 0.95 sec to 1.21sec and 0.82 respectively, also no changes were experienced in overshoot and steady-state error as showed in Figure 8, Figure 9 and Table 3. Moreover, IAE and ISE were used as the performance indexes of the proposed control schemes and based on the numerical values recorded in Table 2, the performances of the proposed novel hybrid control scheme is excellent as the less and negligible error was recorded.

### Table 2. Performance indexes

<table>
<thead>
<tr>
<th>Payload Mass (g)</th>
<th>IAE (rad)</th>
<th>ISE (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0g</td>
<td>$4.822x10^{-7}$</td>
<td>$5.012x10^{-7}$</td>
</tr>
<tr>
<td>30g</td>
<td>$5.45x10^{-7}$</td>
<td>$5.72x10^{-7}$</td>
</tr>
<tr>
<td>50g</td>
<td>$5.83x10^{-7}$</td>
<td>$6.145x10^{-7}$</td>
</tr>
</tbody>
</table>

### Table 3. Time response analysis

<table>
<thead>
<tr>
<th>Payload Mass (g)</th>
<th>Max. Overshoot (%)</th>
<th>Settling Time (s)</th>
<th>Rise Time (s)</th>
<th>Tip Deflection (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0g</td>
<td>0.00</td>
<td>1.3</td>
<td>0.95</td>
<td>$2.2x10^{-3}$</td>
</tr>
<tr>
<td>30g</td>
<td>0.00</td>
<td>1.25</td>
<td>0.85</td>
<td>$3.3x10^{-3}$</td>
</tr>
<tr>
<td>50g</td>
<td>0.00</td>
<td>1.21</td>
<td>0.82</td>
<td>$3.9x10^{-3}$</td>
</tr>
</tbody>
</table>

5. **CONCLUSION**

In this paper, tip deflections and precise hub angle position control of single-link flexible manipulators have been proposed. An OBCS filter was designed using the output of the single-link flexible manipulator to suppress tip deflections and then LMI closed-loop control scheme was incorporated for precise hub angle tracking. The robustness of this hybrid control scheme was tested by changing the mass of the payload from 0g, 30g, and 50g. The IAE, ISE, and TRA were used as the performance indexes. Based on the simulation results, tip deflections and residual vibrations were well suppressed and precise hub angle positioning was achieved. The control schemes are robust to payload variation, hence recommended for real-time implementation.

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